A Stochastic, Dynamic Model for Optimizing Home Video Release

Franco Berbeglia*, Timothy Derdenger, Sridhar Tayur

Tepper School of Business, Carnegie Mellon University, Pittsburgh PA 15213

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Abstract

We study how innovations in the home video viewing experience affect the optimal home video window—the time duration between theater market exit (a random variable) and home video release—in the movie industry. We specifically analyze the case when there are two home video products with different technological qualities: DVDs and Blu-rays. A dynamic discrete choice model connects theater and home video markets through the theatrical performance, window duration, and discounted value of waiting. Different from extant literature, our consumers are forward looking and may postpone their purchases for higher expected utilities in the future. Furthermore, the three markets are intertwined, so changing the release date for a home video of one technological quality will have an impact on the box office revenue, which will in turn impact home videos of all other technological qualities. We estimate the model parameters using panel data (for action movies) on the weekly level for home videos and box office. We conduct a counterfactual analysis in which the home video windows for DVDs and Blu-rays are jointly optimized. We find that an immediate after-theater release of lower technological quality home videos (DVDs) combined with a 5-week delay on higher technological quality home videos (Blu-rays) is optimal. We attribute this result to consumer heterogeneity, where there is greater substitution between higher technological quality home videos and theaters. Our methods can also be applied to the "streaming versus theater" trade-off analysis.

1 Introduction

Home video release timing is among the most important decisions for movie distribution. As a market segmentation strategy, it separates two different consumer experiences: theaters and home

*fberbegl@andrew.cmu.edu. Corresponding author.

videos. For the last few decades, the home video window—the time between theater market exit and home video release—has been decreasing. At the same time, the technological quality of the home video experience has improved, from VHS in the 1990s to 4K Ultra HD today. This poses a question for the movie industry: is it optimal to further decrease the home video window as the technological quality of home videos increases, or should the window be increased? Furthermore, as several technologies of home videos are available at the same time, for example DVD and Blu-ray during the 2010's, which technological quality home video should be released first, and how far ahead of the other, in order to maximize studio revenue?

Advertising plays an important role in movie distribution. In general, 80% of the advertising budget is spent during theatrical release, while the remaining 20% is left for the home video release. This generates incentives for studios to release their home videos early, as the advertising expenditure in theaters will have a stronger spillover effect on home video consumption. However, at the same time, some consumers may be willing to delay the box office purchase decision to the home video market if the wait is not too long, reducing the box office revenue. This reduction in box-office receipts impacts the home video market as well, as it is well known that the theatrical revenue serves as a quality signal that drives home video demand. This trade-off illustrates the intertwining factors that affect the optimal choice for the release of home videos; this choice might also differ depending the varying technological qualities of home videos. The theatrical financial performance, and the associated run length, are stochastic, adding to the complexity of the setting.

In the past few years, we have seen several efforts from distributors to shrink the home video window. With the start of the COVID-19 pandemic in December 2019 and the limited market power of theaters, these efforts expanded. In July 2020 AMC (the world's largest movie theater chain) and Universal struck an agreement to release home videos just 17 days after theatrical release (Watson 2020). Similar agreements were implemented by other studios, with Warner Bros. announcing it will release all of its 2021 films simultaneously in theaters and on the HBO Max platform (Schwartzel and Flint 2020). Recently, ViacomCBS launched its own streaming platform, Paramount+, which will release Paramount and some MGM titles just 45 days after theatrical release (Kit 2021). These changes were made in the midst of a market dominanted by home viewing; thus, it remains uncertain whether theaters will regain their influence on the market.

This paper expands the growing set of examples that use structural estimation in empirical operations management, see Terwiesch et al. (2020). Structural estimation is a technique that estimates parameters of theoretical economic models, where these parameters are estimated using methods such as generalized method of moments or maximum likelihood. A structural model is complex and often involves sequential decision-making under uncertainty or strategic environments where beliefs about other agents' actions matter. The benefit of structural models are the powerful counterfactual analyses they may provide, wherein our case these pertain to the supply side and the optimization of the home video window (e.g. the revenue impact of delaying or advancing home video release). Specifically, this paper develops a dynamic structural model for the box office market and the home video market, consisting of two technological qualities, DVDs and Blu-rays[1]. The model is built around release timing strategies, and it is able to quantify the change in revenue associated with advancing or delaying home video releases. In order to do this, modeling consumer forward looking behavior is critical. We model the home video market as an infinite horizon model for each movie and technological quality, where consumers decide whether to buy the disc, or to delay their purchase in each period. We model the box office market for each movie as a dynamic program with a finite horizon. During each period, consumers face the decision whether to buy a ticket or wait for the next period. The terminal continuation value of the box office market is set as the discounted value of the home video market, which depends on whether the consumer owns a DVD or Blu-ray player.

Consumer behavior drives the movie industry and thus it is vital that consumer demand is properly modeled and estimated to assess a firm's supply side decision. Moreover, understanding how the industry functions and which decisions studios and theaters make is equally important to correctly model both the demand and supply sides. For instance, the time a movie remains in theaters is not a decision the studios make unilaterally, but one that theaters decide, depending on film performance. Thus, consumers in the box office market do not know the horizon of such a market, and have to form expectations about its duration in each period. Under our setting, consumers form dynamic expectations about the time a movie remains in theaters. These expectations depend on movie performance and time since release. Consumers are also forward looking in the evolution of the movie quality over time, the home video market value, and the home video window, all of which determine the terminal continuation value of the box office market. A major technical effort in this paper is the development and estimation of the demand for all the three markets using a structural model.

With the demand estimates available for the box office, DVD, and Blu-ray markets, we are able

[1]This can be seen as a proxy for Full HD and 4K videos today. It is simple to add other home videos as the data becomes available

to perform a series of counterfactual analyses to determine the optimal home video window. First, we estimate the impact of modifying the *simultaneous* release of DVDs and Blu-rays, which was the industry practice at the time of data collection. We leave everything constant, while adjusting for the advertising build-up (as having a shorter home video window increases the advertising build-up coming from theaters) and the box office revenue signal. We find that it is optimal to set the DVD and Blu-ray release 2.3 weeks after theatrical exit to achieve a 4.47% increase in studio revenue[2], which has little impact for theaters. Second, we estimate the impact of *separately* modifying the home video window for DVDs and Blu-rays. This allows the studio to further exploit market segmentation, as consumers express heterogeneous preferences depending on the technological quality of the home video player they possess. We find that the optimal strategy is to release DVDs within a week of theatrical exit, but to delay Blu-rays about 5.15 weeks from theatrical exit. This strategy increases studio revenue by 5.36% with respect to the data, and again, the impact to theaters is minimal.

Let us provide the insights behind these results: They are driven by the different substitution patterns that home videos of varied technological quality present to theaters. Lower technological quality home videos, such as DVDs, have a low ex-ante value function for their market compared to the market for higher technological quality home videos, such as Blu-rays. This makes DVD owners less likely to delay the box office purchase decision compared to Blu-ray player owners when shortening the home video window. Thus, the studios can reap all the benefits of the advertising spillover effect from theaters while posing minimal competition to theaters. The trade-off is more nuanced for Blu-rays. Shortening their home video window will increase substitution from theaters, and will further impact the DVD market, as the quality signal coming from box office revenue is reduced. This result captures consumer heterogeneity in the box office market; Blu-ray player owners value picture quality above everything else, whereas DVD player owners value timing and pricing above picture quality. This allows the studios to perform market segmentation strategies on releases to boost revenue.

The technical contributions of our paper include the modeling and estimation of (a) a finite horizon, dynamic, discrete choice model, with a horizon length that follows a path dependent distribution, and (b) different technological quality home videos (DVD and Blu-ray) that compete with the box office market for each particular movie. These techniques can be used in other markets

[2]Studio revenue captures all of the home video revenue, and a share of the box office revenue that comes from imposing a standard theater-distributor contract.

in the entertainment industry such as (a) video on demand sales and rentals, and (b) shows with multiple episodes. Beyond entertainment, these methods are applicable in fast fashion, where consumer trends are likewise unknown in a path dependent manner, and can have continuation value after the season.

The remainder of this paper is organized as follows: Section 2 presents the related literature, Section 3 describes the industry and shows the main attributes of the dataset used, Section 4 presents the structural model including consumer utility specifications and the consumers' optimal purchase problem, Section 5 explains the estimation and identification procedure for the model parameters, Section 6 shows and describes the parameter estimates of the model, and Section 7 presents the counterfactual analysis on the home video windows. We conclude in Section 8.

2 Literature Review

Below we divide the relevant literature into two distinct areas: structural estimation in empirical operations management and movie related research–including movie release timing.

2.1 Structural Estimation in Empirical Operations Management

The use of structural estimation has been growing in empirical operations management research. Terwiesch et al. (2020) presents a detailed summary of the current papers, presenting the research questions, key parameters, and methods.

The majority of operations papers that use structural estimation address supply side related questions. To do so, they estimate consumer demand using methodologies from Rust (1987), Berry (1995), and/or Nevo (2000), for input into the supply side model. Rust (1987) develops a regenerative optimal stopping problem for bus engine replacement, Berry et al. (1995) develops techniques for empirically analyzing demand and supply in markets with differentiated products, and obtain demand and cost estimates, whereas Nevo (2000) develops a practitioner's guide for estimating random-coefficients logit models. In our case, we base our demand model on the framework of Gowrisankaran and Rysman (2012), which combines Berry et al. (1995) for modeling the discrete choice demand and Rust (1987) for the optimal stopping decisions to estimate a dynamic discrete choice demand model. We also employ the works of Nevo (2000) to obtain the final fixed-effect estimates, and Derdenger (2014) to connect the box office and home video markets.

Recently, there has been several structural estimation papers in the service management space.

Akşin et al. (2013) studies the decision-making process of callers in call centers as an optimal stopping problem based on Rust (1987), where the utility of a caller is modeled as a function of her waiting cost and reward for service. After the utility parameters are estimated, the authors conduct a counterfactual analysis to assess the impact of changes in the service discipline. This presents several similarities with our paper, where the decision-making for customers is also modeled as an optimal stopping problem for each market, and, furthermore, we use counterfactual analysis to assess the impact of changes in the home video window on movie revenue. In a similar fashion, Hu et al. (2021) study the mechanism of customer retrials in call centers through a dynamic structural model in order to provide economically feasible solutions to reduce retrials. Guajardo et al. (2016) use the framework from Nevo (2000) to study the impact of service attributes (warranty length, after-sales service quality) on consumer demand in the U.S. automobile industry. Allon et al. (2011) use the framework from Berry et al. (1995) to study the consumer cost of waiting in the fast food industry. Finally, Li et al. (2014) use a structural model to analyze whether consumers are strategic in their decisions of executing or delaying flight ticket purchases. Similarly to our paper, Guajardo et al. (2016), Allon et al. (2011), and Li et al. (2014) all use the generalized method of moments (GMM) to finalize their respective estimation procedures.

2.2 Movie Release Timing and Other Relevant Research

The problem of finding the optimal inter-release times of sequentially released products has been widely studied in several industries. Moorthy and Png (1992) analyze the optimal timing and quality of sequential product releases. Lehmann and Weinberg (2000) analyze the problem of demand cannibalization between box office and home videos while trying to reap gains as quickly as possible.

We extended this general line of research in several dimensions. First, we capture box office consumer forward looking behavior and allow advertising expenditure to accumulate over time. The latter increases the incentives to shorten inter-release times and is a major driver for shorter inter-release times. We additionally focus on how the optimal inter-release times change as the technological quality of home videos increases from DVDs to Blu-rays, while creating heterogeneous consumer preferences between these products. Another innovation of our model is that consumers form expectations about the evolution of movie quality during each market, as well as on the time a movie remains in theaters. We develop a discrete hazard model to asses the probability distribution of the remaining time in theaters of a movie that is dependent on the time since release and performance thus far.

Other relevant work on movie release timing may be found in the study of the trade-off between seasonality and freshness for DVD; see Mukherjee and Kadiyali (2018). August et al. (2015) and Calzada and Valletti (2012) analyze, through theoretical models in different settings, the conditions under which day-and-date, direct-to-video, or delayed home video releases are optimal release timing strategies for the movie industry.

Finally, there are numerous papers that empirically analyze demand in the movie industry. This includes Eliashberg and Shugan (1997), Eliashberg et al. (2000), Elberse (2007), Eliashberg et al. (2014), and Packard et al. (2016), who study the impact of critics' reviews, star actors, networks of cast and crew, and opening weekend box office on overall revenue performance. Lehmann and Weinberg (2000), Elberse and Anand (2007), and Rao et al. (2017) analyze the impact of advertising on box office revenue. However, these papers only do so in a non-structural reduce form manner.

3 Industry Setting and Data

The motion picture industry comprises three stages: production, distribution and exhibition. The production stage consists of the development of a motion picture and is a creative process with important economic implications for the parties involved. The process usually begins with an idea, concept, or true event, which a writer captures in a screenplay. If a producer is interested in the screenplay, she may sign an option agreement with the writer, which gives the producer the possibility of purchasing the complete screenplay, and provides an upfront payment for the writer. Substantial financing is needed to begin production, a cost that is lowered when the producer is affiliated with a studio. Upon the signature of a studio contract, the producer gives up several rights, including sequels, spin-offs, and merchandising. At the same time, the producer increases her chances of obtaining bank loans and securing favorable distribution and exhibition deals. These contracts benefit the studios, as they provide a constant inflow of products from successful films. Many producers face financing issues when they cannot reach a deal with a studio; in such cases the studio must obtain financing from other sources, which is difficult when no distribution deals are guaranteed (Vogel 2014).

The distribution stage begins once a movie has completed production, and it includes the distribution to theaters, home video markets, as well as the marketing activities in each market where the movie is released. Distributors face a wide range of decisions in this stage, including when to release the movie in each channel and the advertising strategy for the motion picture. Among distributors there is a clear distinction between major and independent firms. The major distributors—usually referred to as "The Big Six"—include Paramount, Sony, Twentieth Century Fox, Universal, Walt Disney, and Warner Bros. These studios produce, finance, and distribute their own movies. At the same time, they also finance and distribute films produced by independent film makers who are associated with the studio. Ensuring a strong US theatrical box-office gross is very important for these studios, because it is a performance metric to indicate sales potential in other distribution channels such as global theatrical, home video, and pay television (Eliashberg et al. 2006). Simultaneously, the growing importance of non-theatrical channels as a source of revenue is generating incentives for the studios to reduce the time between theatrical and non-theatrical releases. Figure 1 shows the evolution of the average DVD release windows, (time between theatrical and DVD releases), for major studios and years. This reduction in non-theatrical windows poses several questions, which we address in this paper. One of them was proposed by Eliashberg et al. (2006): "To what extent are theatrical and nontheatrical windows substitutes or complements (i.e., either negatively or positively affecting each other's revenue potential)? For example, does the availability of DVDs deter people from going to the theater?". Building upon this question, we analyze the optimal non-theatrical window across different technological quality home videos, which present different substitution patters with theaters.

In the exhibition stage, major studios have limited control. The contractual agreements between exhibitors and distributors involve a minimum playing time, as well as terms on how the box office revenue is shared between the parties involved. Beyond these, it is up to each individual theater to control the total playing time for each movie (beyond that set minimum). Studios generate a strong buzz prior to and during their theatrical release—combining advertising, word of mouth, and media attention—which drives demand for the motion picture in other distribution channels (Eliashberg et al. 2006).

3.1 Data

We estimate our model using panel data for box office, DVD, and Blu-ray sales. The data are at the weekly level for all panels. For each week that a movie is in the box office, our data include revenue ($), ticket sales, number of theaters, and other characteristics. For each movie and week in a home video environment, our data include unit sales, revenue ($), and average price. We observe 149 movies, across box office, DVD and Blu-ray, with observations between the years 2009 and

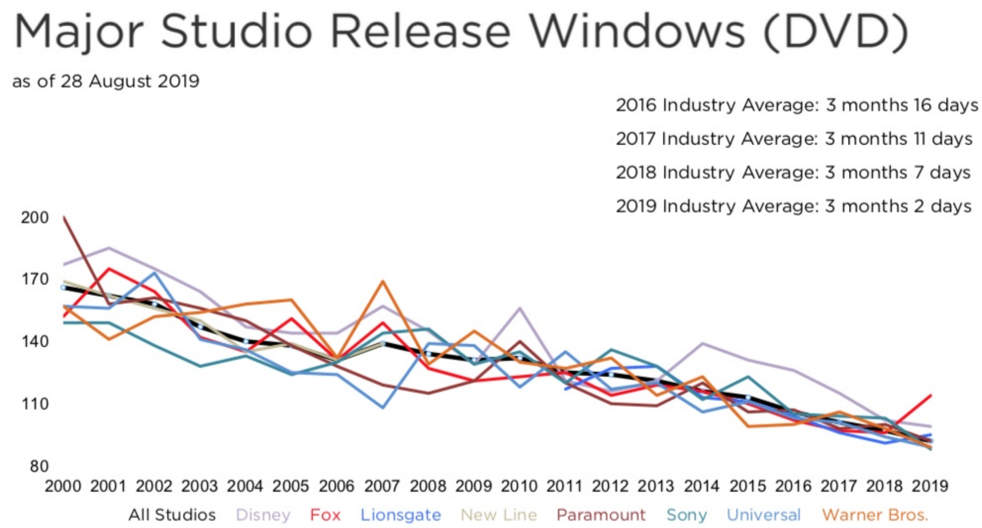

Figure 1: Evolution of the number of days between theatrical and DVD release for major studios. Source: https://www.natoonline.org.

2018. Out of those 149 movies, only 113 had Blu-ray releases, while all of them had a DVD release. We also obtain advertising expenditure data for each movie at the monthly level from Ad$pender. For each movie we also have information on its characteristics, such as distributor, domestic box office revenue, production budget, home window lengths and release dates. We also scraped poster images from www.themoviedb.org and applied color theory on them to extract hue information; the procedure is described in section 5.3.4. We create market shares by dividing the unit sales by the number of consumers, for the box office, or number of DVD/Blu-ray player owners for the home videos. We use data from the U.S. Census Bureau to get the number of consumers within each movie rating per year, and data from the Consumer Electronic Association to get information on DVD/Blu-ray player ownership. These data are interpolated to the monthly level, assuming a linear growth rate. We adjust inflation in all revenues to January 2019 dollars using data from the Bureau of Labor Statistics. To create our final dataset, we remove box office panel weeks with fewer than 100 theaters and we define the start of the home window as the week in which a movie is in less than 100 theaters. The final dataset consists of 1,797 observations across 149 movies for the box office, 44,800 for DVDs (across the same 149 movies), and 25,373 observations for Blu-rays across a subset of 113 movies. Table 1 presents summary statistics of the production budget, advertising expenditure, and revenue per medium of the final dataset, and Table 2 presents a summary of the home windows and time in theaters. It is important to note that the difference between the DVD and Blu-ray home video windows is due to the different sample sizes, and not due to different home video release dates for the same movie, as the industry practice has been one of simultaneous release of DVD and Blu-Ray when both were made available.

	Mean	**Median**	**Standard Deviation**
Production budget	$ 121,775,960	$99,979,420	$ 77,255,980
Domestic box office revenue	$ 122,169,272	$ 82,897,417	$ 106,867,804
Total DVD revenue	$ 37,480,615	$ 26,050,639	$ 42,840,121
Total Blu-ray revenue	$ 22,478,904	$ 16,729,886	$ 19,335,287
Total advertising expenditure	$ 27,612,363	$ 27,170,200	$ 11,091,510

Table 1: Data summary on revenues, advertising and budgets.

	Mean	**Median**	**Standard Deviation**	**Maximum**	**Minimum**
DVD home window (weeks)	6.6951	6.2857	3.6906	16.2857	0.2857
Blu-ray home window	6.1871	6.2857	3.6453	16.2857	0.2857
Time in theaters (weeks))	11.4007	11.0000	3.1829	22.0000	6.0000

Table 2: Data summary on timing characteristics.

3.2 Data Analysis

We present reduced form results that show the connection between relevant covariates and revenue. We retrieve estimates through a two stage process: (1) fixed effects regression that includes time dependent characteristics, and (2) retrieval of movie characteristics from movie fixed effects.

Table 3 shows the box office reduced form using panel data on weekly box office ticket sales. It shows the first stage regression on top, and the fixed effect GLS (General Least Squares) retrieval of static movie characteristics at the bottom. The sign of the coefficients is quite intuitive; as a movie ages, the demand for it decreases, while an increase in advertising expenditure yields greater demand[3]. The average box office price per week is not significant, probably due to the low variation

[3]We create advertising covariates following Dubé et al. (2005), we describe the procedure in Section 5.3.3.

of price across both time and titles[4]. The "lag 1st3weeks box revenue" covariate represents the total lagged box office revenue until the current period, or period 3 included, whichever comes first. This shows that if a movie performs well in the first few weeks after release, it will drive up demand for the consecutive weeks, but with diminishing returns, as the quadratic component is negative. The fixed effects regression shows that production budget and time in theaters are major drivers for box office demand. The home window contribution is smaller in magnitude, but presents the expected substitution pattern with a positive sign, which means that box office revenue increases by delaying home video releases.

Table 4 shows the home video reduced form using panel data on home video sales. The top of the table shows the first stage regression in which DVD and Blu-ray weekly sales are regressed against time dependent covariates, and an interaction between them and a Blu-ray indicator variable with movie/medium fixed effects. We then run a fixed effects GLS regression for DVDs and Blu-rays separately on time independent characteristics. In the first stage regression, we controlled for price endogeneity using lagged prices as instrumental variables. We can see that home video demand lowers with age and price. Advertising expenditure has a positive impact on home video sales, but this effect is lower for Blu-rays. The fixed effects GLS regression shows an increasing demand for home videos with an increase in box office opening revenue within the ranges of box office revenues observed. As for the home video window, DVD demand seems to be larger with lower windows, while Blu-rays exhibit concave demand shape with a maximum in around 8 weeks.

These reduced form results suggest several features that are important to embed into a structural model. These include expectations about the home video window and time in theaters during the theatrical market, box office revenue as a signal of movie quality for the home video market, and advertising. Furthermore, the significant coefficients for age in both markets show that freshness is an important factor for viewing.

4 Demand model

We discuss the structural model that captures the relationship between box office ticket sales, home video sales, and the forward-looking behavior of consumers. The box office and home video markets are linked through the home video window, which is the time between theatrical exit and home video release. Shrinking this window leads to higher freshness on the home video market

[4]To control for price endogeneity we used lagged prices as an instrument.

Box office: two-stage reduced form

Time Dependent Variables	**Estimates**
Age (weeks)	-0.8184 (0.0161)***
Age^2 ($weeks^2$)	0.0223 (0.0008)***
Price ($)	-0.6720 (0.5636)
Advertising	0.50933 (0.1361)***
log(lag 1st3weeks box revenue)	0.23651 (0.0474)***
log(lag 1st3weeks box revenue) ^{2}	-0.0128 (0.0027)***
Movie fixed effects	
Month fixed effects	
N	1,797
Time Independent Variables	**Estimates**
Constant	7.3604 (1.3068)***
log(production budget)	0.2551 (0.0343)***
Time in theaters (week)	0.4945 (0.0297)***
Time in $theaters^2$ ($week^2$)	-0.0111 (0.0011)***
Home window (week)	0.0039 (0.0170)
Home $window^2$ ($week^2$)	0.0024 (0.0010)***
Release year fixed effects	
Distributor fixed effects	
Color fixed effects	
N	149

$^{***}p < .01$, $^{**}p < 0.05$, $^{*}p < .1$

Table 3: Reduced form estimates of a two-stage fixed effects model on weekly box office ticket sales.

Home video: two-stage reduced form

Time Dependent Variables	Estimates Home Video	Estimates Blu-ray Indicator
Age (weeks)	-0.0119 (0.0035)***	-0.0866 (0.0263)***
Age^2 (weeks^2)	0.0000 (0.0000)***	-0.1119 (0.0365)***
Price ($)	-0.0182 (0.0012)***	-0.2212 (0.0499)***
Advertising	0.6462 (0.0078)***	-0.2832 (0.0647)***
Movie fixed effects	✓	✓
Month fixed effects	✓	✓
Year fixed effects	✓	✓
N	70,253	25,373
Time Independent Variables	**Estimates DVD Indicator**	**Estimates Blu-ray Indicator**
Constant	33.3590 (0.5997)***	51.2524 (0.9777)***
log(1st3week box revenue)	-3.3500 (0.0705)***	-5.6018 (0.1139)***
$\log(\text{1st3week box revenue})^2$	0.1030 (0.0021)***	0.1709 (0.0033)***
Home window (week)	-0.0233 (0.0053)***	0.0989 (0.0061)***
Home window^2 (week^2)	0.0015 (0.0003)***	-0.0056 (0.0004)***
Release year fixed effects	✓	✓
Distributor fixed effects	✓	✓
Color fixed effects	✓	✓
N	149	113

$^{***}p < .01$, $^{**}p < 0.05$, $^{*}p < .1$

Table 4: Reduced form estimates of a two stage fixed effects model on weekly home video sales.

and a greater spillover of advertising spent for the theatrical market, but it may lead to demand cannibalization from theaters. Consumers in the box office market form expectations about this home video window and decide whether to watch the movie in the current week, or delay their decision to the following one. Consumers form expectations about the evolution of prices, movie quality, and theatrical run time (equivalently, the start of the window). Since we are interested in analyzing the aforementioned trade-off disregarding competition, we consider each movie to be a monopoly.

The timeline in our model is as follows: Consumers own either a DVD or a Blu-ray player when a movie is released in theaters. In each week of the theatrical market, consumers decide whether or not to buy a box office ticket. They continue to make such a decision until they elect to watch the movie, or wait until the theatrical run time is over. After the theatrical run is over, we enter the home video window. During this period, consumers are not able to watch the movie in theaters, nor buy a home video for this movie. Then, the home video is released and we enter the video time. In each period (week) of the home video market, consumers decide whether or not they will purchase their respective discs (DVD or Blu-ray). This is the situation where both DVD and Blu-ray are made available simultaneously. They continue to make such a decision in each period until they elect to purchase the home video. Figure 2 shows the described timeline of the model. In order to estimate the model involving the discussed forward-looking behavior, we use the frameworks of Gowrisankaran and Rysman (2012) and Derdenger (2014). The former paper embeds consumer expectations about price, movie characteristics, and other (unobservable) factors that might evolve over time. The latter paper is used to link the utilities between the theatrical and home video markets.

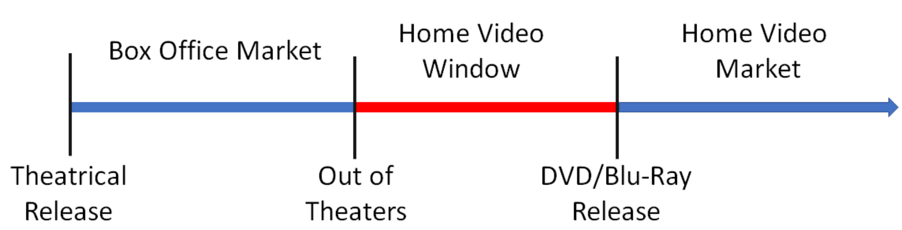

Figure 2: Timeline of movie distribution.

Next, we present the utility specifications for the home video and theatrical markets, first discussing the associated utilities for home videos and then for box office tickets.

4.1 Home video utility

We now outline the utility model for home video consumption. The home video utility specification follows that of Gowrisankaran and Rysman (2012); it is an infinite horizon model. Our specification allows for model parameters to differ between DVDs and Blu-rays so that the expected value of the home market for users who hold a DVD or a Blu-ray player are different. This term plays an important role in the box office market, since it enters through the terminal continuation value of such a finite horizon model.

In the home video market, each consumer decides in each time period t whether or not to purchase a home video for movie/medium j. If consumer i decides to purchase a home video for movie/medium j of technological quality $k \in \{\text{DVD, Blu-ray}\}$[5] in time period t, she obtains a utility given by

$$u^h_{i,j,t} = f^h_{j,t} + \alpha^{p,h}_k p^h_{j,t} + \epsilon^h_{i,j,t}, \tag{1}$$

where $f^h_{j,t}$ is the flow utility from the home video, $p^h_{j,t}$ is the price of the home video, $\alpha^{p,h}_k$ is the price sensitivity for home videos of technological quality k, and $\epsilon^h_{i,j,t}$ is an idiosyncratic shock which we assume to be the realization of a Type-1 Extreme Value distribution that is independent and identically distributed across consumers, products, and time periods.[6]

A consumer who does not purchase a movie/medium j of technological quality $k \in \{\text{DVD, Blu-ray}\}$ in period t receives

$$u^h_{0,j,t} = \beta \mathbb{E}[V^h_k(\Omega^h_{j,t+1})|\Omega^h_{j,t}] + \epsilon^h_{0,j,t}, \tag{2}$$

where $\Omega^h_{j,t}$, is the industry state of movie/medium j at period t, involving the flow utility, price disutility, and all history factors that influence the home video's future attributes. Finally, $V^h_k(\Omega^h_{j,t})$ represents the value of having the purchase possibility of home video j of technological quality $k \in \{\text{DVD, Blu-ray}\}$, when the state of home video j is $\Omega^h_{j,t}$, while β is the weekly discount factor.

The home video flow utilities depend on movie characteristics with the following relation:

$$f^h_{j,t} = \alpha^{fe,h}_j + \alpha^{x,h}_k x^h_{j,t} + \xi^h_{j,t}, \tag{3}$$

where $x^h_{j,t}$ are time dependent observable movie characteristics (for DVD and Blu-ray), such as age, advertising expenditure, price, month, and year; $\alpha^{fe,h}_j$ are movie/medium fixed effects; $\alpha^{x,h}_k$ are

[5] Movie/medium j includes information about the technological quality of the home video, so we defer from adding subscript k to denote such technological quality when subscript j is present.

[6] Let superscript h and b denote utility specification for home video and box office, respectively.

time dependent characteristics coefficients that depend on whether the technological quality of the home video is a DVD or a Blu-ray; and $\xi^h_{j,t}$ are unobservable characteristics that vary both over time and across movies. It is important to remark that we don't explicitly express the dependence of $f^h_{j,t}$ on k since j (movie/medium) already includes that information.

The estimation of the model parameters involves a second stage in which the movie medium fixed effects are regressed on time fixed movie characteristics, following Nevo (2000). Examples of movie characteristics involve opening box office revenue, distributor, the home window length, poster colors and release year.

4.2 Box office utility

Unlike the home video market, the box office market has a finite horizon. The consumer type is denoted by the subscript $k \in \{\text{DVD, Blu-ray}\}$, which provides a source of heterogeneity that depends on the technological quality of the home video player owned. This heterogeneity enters exclusively in the outside option and not in the purchase utility, as we assume that the box office ticket purchase continuation value is zero for both consumer types [7]. In each period t, consumer i considers whether or not to watch a particular movie j. Once a consumer watches movie j, she exits the box office market for movie j. If consumer i decides to purchase a box office ticket for movie j in time period t, she obtains utility given by

$$\begin{aligned} u^b_{i,j,t} &= \phi \cdot (\tilde{f}^b_{j,t} + \tilde{\alpha}^{p,b} p^b_{j,t}) + \epsilon^b_{i,j,t} \\ &= f^b_{j,t} + \alpha^{p,b} p^b_{j,t} + \epsilon^b_{i,j,t}, \end{aligned} \tag{4}$$

where $f^b_{j,t}$ is the flow utility from observable and unobservable box office characteristics, $p^b_{j,t}$ is the price of the box office ticket, and $\epsilon^b_{i,j,t}$ is an idiosyncratic shock. The parameter ϕ is a scaling parameter that permits the comparison between the home video and box office markets; it serves as a utility normalization factor, since we forced the error terms for both markets to have the same variance.

A consumer who does not watch movie j in period t and who owns a home video player of technological quality $k \in \{\text{DVD, Blu-ray}\}$ receives

$$u^b_{0,k,j,t} = \beta \mathbb{E}[V^b_k(\Omega^b_{j,t+1}) | \Omega^b_{j,t}] + \epsilon^b_{0,j,t}, \tag{5}$$

[7] This can be transformed into a setting in which consumers obtain a flow utility in each period and the discounted sum of these utilities is equal to the purchase utility in our model

where $\Omega^b_{j,t}$, is the industry state of movie j in theaters at period t, involving the flow utility, price disutility, and all history factors that influence the movie's future attributes. (For instance, revenue and time since release, number of theaters available, etc. are important to forming expectations about the last period of the market). Finally, $V^b_k(\Omega^b_{j,t})$ represents the value of having a box office ticket purchase possibility when the state of movie j is $\Omega^b_{j,t}$ and the consumer has a home video player of technological quality $k \in \{\text{DVD, Blu-ray}\}$. Note that the outside option depends on j since we are modeling each movie as a monopoly, and the continuation value of delaying the box office purchase decision is different for each movie.

The box office flow utilities depend on the movie features with the following relation:

$$f^b_{j,t} = \alpha^{fe,b}_j + \alpha^{x,b} x^b_{j,t} + \xi^b_{j,t}, \tag{6}$$

where $x^b_{j,t}$ are observable box office movie characteristics that vary over time, $\alpha^{x,b}$ are the coefficients of observable time dependent movie characteristics, $\alpha^{fe,b}_j$ are movie fixed effects, and $\xi^b_{j,t}$ are the unobservable components of the flow utility that vary both over time, and across movies. Examples of time-dependent box office movie characteristics include advertising expenditure, average price, month, age, and performance up to current period. Similarly to the home video market, the estimation of the model parameters involves a second stage in which fixed effects are regressed on time invariant movie characteristics. Examples of movie characteristics involve distributor, production budget, and release year. Note that in (4) and (6) we assume that the consumer purchase utility specification about movies in theaters does not depend on the type of home video owned (DVD or Blu-ray), but this heterogeneity enters in (5), the outside option.

This utility specification generates a challenge. The box office market has a finite horizon; thus, the model has to embed consumer's expectations about the market's horizon, the home window length, and the value of the home video market in order to quantify continuation values. To embed consumer's expectations we must make assumptions on what factors affect the movie's industry state $\Omega^b_{j,t}$. This is analyzed within the consumer's problem, in section 4.3.2.

4.3 Consumer's problem

We now outline the consumer's decision process, which incorporates the forward looking behavior about the evolution of the movie industry states $\Omega^b_{j,t}$. We begin by describing the decision process for the home video market as it is independent of the value functions of the box office market. We then describe the finite horizon model of the box office market, where the home video value enters through the terminal continuation value.

4.3.1 Home video market

The home video market consists of two separate markets differentiated by technological quality. We distinguish between them with subscript $k \in \{\text{DVD, Blu-ray}\}$. Each of them can be seen as an optimal stopping problem with an infinite horizon. In each period, consumers have the possibility to purchase their respective home video disc, or to wait. Following Equations (1) and (2), the value function prior the realization of $\vec{\epsilon}^h_{j,t} \doteq (\epsilon^h_{i,j,t}, \epsilon^h_{0,j,t})$ for a consumer who owns home video player $k \in \{\text{DVD, Blu-ray}\}$ can be written as

$$V^h_k(\Omega^h_{j,t}) = \int \max\left\{f^h_{j,t} + \alpha^{p,h}_k p^h_{j,t} + \epsilon^h_{i,j,t}, \beta\mathbb{E}[V^h_k(\Omega^h_{j,t+1})|\Omega^h_{j,t}] + \epsilon^h_{0,j,t}\right\} g_\epsilon(\vec{\epsilon}^h_{j,t}) d\vec{\epsilon}^h_{j,t}, \tag{7}$$

where $g_\epsilon(\cdot)$ is the probability density function of $\vec{\epsilon}^h_{j,t}$. From (7), the first element of the max operator indicates the purchase utility, while the second indicates the expected discounted value of delaying the purchase decision to the next period.

We proceed by using the aggregation properties of the extreme value distribution to express (7) in a simpler form, and then we make assumptions on how consumers form expectations about future movie industry states. Specifically, we can write

$$V^h_k(\Omega^h_{j,t}) = \ln\Big(\exp(\delta^h_{j,t}) + \exp(\beta\mathbb{E}[V^h_k(\Omega^h_{j,t+1})|\Omega^h_{j,t}])\Big), \tag{8}$$

where

$$\delta^h_{j,t} = f^h_{j,t} + \alpha^{p,h} p^h_{j,t}, \tag{9}$$

the *logit inclusive value*, is defined as the ex-ante present discounted lifetime value of buying a home video at period t, as opposed to waiting for the next period. We make the assumption that consumers consider only the current value of $\delta^h_{j,t}$ to form the expectations of the evolution of V^h to the next period. So the history of the past values of $\delta^h_{j,t}$ does not matter. Thus, we have

$$V^h_k(\delta^h_{j,t}) = \ln\Big(\exp(\delta^h_{j,t}) + \exp(\beta\mathbb{E}[V^h_k(\delta^h_{j,t+1})|\delta^h_{j,t}])\Big). \tag{10}$$

We employ rational expectations for future values of $\delta^h_{j,t}$ by imposing a simple linear autoregressive specification:

$$\delta^h_{j,t+1} = \nu^h_{1,k} + \nu^h_{2,k}\delta^h_{j,t} + \eta^h_{j,t+1,k}, \tag{11}$$

where $\eta^h_{j,t+1}$ is normally distributed with zero mean and unobserved at time t, while $\nu^h_{1,k}$ and $\nu^h_{2,k}$ are general parameters to be estimated for $k \in$ {DVD, Blu-ray}. This assumption ensures that consumers are on average correct about the movie quality evolution. It is important to remark that the optimal consumer decisions, given a movie state $\delta^h_{j,t}$, will depend on the joint solution of the Bellman equation (10) and the movie state regression (11).

Once these two equations are solved, we can obtain the value functions $V^h_k(\delta^h_{j,t})$, and we then use them to estimate the individual purchase probabilities. The movie/medium j purchasing probability for a consumer at period t is given as a function of $\delta^h_{j,t}$, and is

$$\hat{s}^h_{j,t}(\delta^h_t) = \frac{\exp(\delta^h_{j,t})}{\exp(V^h_k(\delta^h_{j,t}))}. \tag{12}$$

4.3.2 Box office market

The box office market differs from the home video market in a few salient ways. First, the market has a finite horizon that is unknown by the consumers, and second, the discounted home video value enters the box office model through the terminal continuation value of this unknown horizon. This allows consumers to substitute between the box office market and the home video market, if this terminal continuation value is large enough. Recall that the home video value differs between the technological quality of the home video, that is, DVD or Blu-ray, which may drive different substitution patterns between the different technological quality markets and the box office.

In order to model the unknown finite horizon, we use the time and revenue since release as drivers for a distribution of possible horizons. We use the data in order to fit a discrete hazard model that gives the probability for each possible horizon. This captures the endogeneity of theatrical runtime, where each theater decides based on performance whether to keep a movie in theaters or not. This procedure is described in Section 4.3.3

Following Equations (4) and (5) and using the aggregation properties of the extreme value distribution, similarly to how we arrived at Equation (8) in the home video market, we obtain the following value function equation for $V^b_{k,|T}$, the box office value function at time period t conditional on having a horizon at T and owning a home video player of type $k \in$ {DVD, Blu-ray}:

$$V^b_{t,k|T}(\Omega^b_{j,t}) = \ln\Big(\exp(\delta^b_{j,t}) + \exp(\beta\mathbb{E}[V^b_{t+1,k|T}(\Omega^b_{j,t+1})|\Omega^b_{j,t}])\Big) \quad \forall t = 1,\dots,T-1, \tag{13}$$

where

$$\delta^b_{j,t} = f^b_{j,t} + \alpha^{p,b}_k p^b_{j,t}, \tag{14}$$

$\Omega_{j,t}^b$ is the state of the industry of movie j at time t, and T is the unknown horizon of the market, following a probability distribution that depends on t and the revenue since release. Finally, the terminal value function is set using expectations on the home video window ($NT_{k,j}$), and the home video value of movie j with technological quality k ($V_{k,j}^h$):

$$V_{T,k|T}^b(\Omega_{j,T}^b) = \ln\Big(\exp(\delta_{j,T}^b) + \exp(\mathbb{E}[\beta^{NT_{k,j}} V_{k,j}^h])\Big). \tag{15}$$

Since the horizon of the problem is unknown, consumers form a probability distribution over probable horizons, $g_T(\Omega_{j,t}^b)$, which depends on the industry state of the movie $\Omega_{j,t}^b$ at time t. Then Equations (13) and (15) can be solved for each T and aggregated using the probability distribution over possible horizons for each time period. Finally we have

$$V_{t,k}^b(\Omega_{j,t}^b) = \sum_T V_{t,k|T}^b(\Omega_{j,T}^b) g_T(\Omega_{j,t}^b) \quad \forall t = 1, \dots, T, \text{ and } k \in \{\text{DVD, Blu-ray}\}, \tag{16}$$

where the probability distribution over values of T is created from data using a hazard model that depends on the revenue and weeks since release. Subsection 4.3.3 provides details on this procedure.

We assume that consumers consider only the current value of $\delta_{j,t}^b$, the time since release, t, and the revenue since release, r_t, to form expectations about the future values of purchase decisions. Then our state variables can be assumed to be (t, r_t, and $\delta_{j,t}^b$), since time (t) and revenue since release (r_t) are important to determining the probability distribution over possible horizons. Then, Equations (13), (15), and (16) can be rewritten as

$$V_{t,k|T}^b(\delta_{j,t}^b) = \ln\Big(\exp(\delta_{j,t}^b) + \exp(\beta\mathbb{E}[V_{t+1,k|T}^b(\delta_{j,t+1}^b)|\delta_{j,t}^b]\Big) \quad \forall t = 1, \dots, T-1, \forall k \in \{\text{DVD, Blu-ray}\} \tag{17}$$

$$V_{T,k|T}^b(\delta_{j,T}^b) = \ln\Big(\exp(\delta_{j,T}^b) + \exp(\mathbb{E}[\beta^{NT_{k,j}} V_{k,j}^h])\Big), \text{ and} \tag{18}$$

$$V_{t,k}^b(\delta_{j,t}^b, r_t) = \textstyle\sum_T V_{t,k|T}^b(\delta_{j,t}^b) g_T(t, r_t) \quad \forall t = 1, \dots, T, \text{ and } k \in \{\text{DVD, Blu-ray}\}. \tag{19}$$

Once again, we employ rational expectations for future values of δ^b, by imposing a linear autoregressive specification:

$$\delta_{j,t+1}^h = \nu_1^b + \nu_2^b \delta_{j,t}^h + \eta_{j,t+1}^b, \tag{20}$$

where $\eta_{j,t+1}^b$ is normally distributed with zero mean and unobserved at time t, while ν_1^b and ν_2^b are general parameters for all movies to be estimated. This assumption ensures that consumers are on average correct about the movie quality evolution.

We assume that consumer expectations about the home video window in (15) are based on perfect foresight. We do so because if we did not use perfect foresight, we would need to model consumer beliefs, and when we ran a counterfactual analysis on these home video windows, they would not be consistent with the estimated beliefs. By using perfect foresight, we ensure that model beliefs are consistent with our optimized counterfactual home video window lengths.

Note that equations (17), (18), (19), and (20) must be solved jointly. This is because a change in the Bellman equation will yield different ν_1^b and ν_2^b coefficients, which will impact the Bellman equations. Specifically, these equations need to be solved twice, one time for DVD player owners, and another time for Blu-ray player owners. The difference between these two comes in the terminal continuation values used in (15)[8]. The fixed point will depend on the continuation value used, but we refrain from using it as a variable for V_t^b. Note that both consumer types have the same quality vector, $\delta_{j,t}^b$, since the difference between types lies in the outside option.

Once these equations are solved, we can compute the individual purchase probabilities for each consumer segment. For simplicity, we now refrain from using k to identify consumer types and we use the suprascript $b-dvd$ for DVD player owners and $b-blu$ for Blu-ray player owners instead. The probabilities that a DVD and a Blu-ray player owner purchase a ticket for movie j in period t is given by

$$\hat{s}_{jt}^{b-dvd}(\delta_{j,t}^b, r_t) = \frac{\exp(\delta_{j,t}^b)}{\exp(V_t^{b-dvd}(\delta_{j,t}^b, r_t))} \text{ and } \hat{s}_{jt}^{b-blu}(\delta_{j,t}^b, r_t) = \frac{\exp(\delta_{j,t}^b)}{\exp(V_t^{b-blu}(\delta_{j,t}^b, r_t))}, \tag{21}$$

respectively. And based on the remaining weight of Blu-ray player owners for movie j at time period t, $w_{j,t}^{b-blu}$, we can compute the purchase probability of a random consumer:

$$\hat{s}_{jt}^b(\delta_{jt}^b) = (1 - w_{j,t}^{b-blu})\hat{s}_{jt}^{b-dvd}(\delta_{jt}^b, r_t) + w_{jt}^{b-blu}\hat{s}_{jt}^{b-blu}(\delta_{jt}^b, r_t). \tag{22}$$

4.3.3 Discrete-Time Proportional Hazard Model for time in theaters

Theaters decide when to stop showing a movie, depending on how the movie is performing. To capture this, we build a Discrete-Time Proportional Hazard Model following Cameron and Trivedi (2005) (section 17.10.1). This model gives a probability distribution for the remaining time in theaters, given the number of weeks the movie has been in theaters and the total revenue until then. Let T be the number of weeks a movie is in theaters, and $R(t)$ the box office total revenue by period t; we define the discrete time hazard function

$$\lambda^d(t|R_{t-1}) = Pr[T = t|T \geq t, R_{t-1}], \quad t = 1, \ldots, M,$$

[8]Both the home video value and the home window length may differ between DVDs and Blu-rays.

which denotes the probability t is the last week this movie is in theaters, given it is in theaters in week t and the total revenue until week $t-1$ is R_{t-1}. Then, the associated discrete-time survivor function is

$$S^d(t|R_{t-1}) = Pr[T \geq t|R_{t-1}] = \prod_{s=1}^{t-1} \big(1 - \lambda^d(s|R_{s-1})\big).$$

We can then specialize the continuous PH model to obtain the following expression for the discrete time hazard

$$\lambda^d(t|R_{t-1}) = 1 - \exp\Big(- \exp\left(\ln \lambda_{0t} + \beta_R \log(R_{t-1})\right)\Big), \tag{23}$$

where λ_{0t} for $t = 1, \ldots, M$ and β_R are parameters to be estimated. The associated discrete-time survivor function is

$$S^d(t|R_{t-1}) = \prod_{s=1}^{t-1} \exp\Big(- \exp\left(\ln \lambda_{0t} + \beta_R \log(R_{t-1})\right)\Big).$$

We can finally write the likelihood function as

$$L(\beta_R, \lambda_{01}, \ldots, \lambda_{0M}) = \prod_{i=1}^{N} \Big[\prod_{s=1}^{T_i-1} \exp\Big(- \exp\left(\ln \lambda_{0s} + \beta_R \log(R_{s-1})\right)\Big)\Big] \times \Big(1 - \exp\Big(- \exp\left(\ln \lambda_{0T_i} + \beta_R \log(R_{T_i-1})\right)\Big)\Big)$$

where i refers to each individual movie, and T_i for the time in theaters of such a movie. We can the use the data to maximize this function over the parameters, and obtain our final Discrete-Time Proportional Hazard Model. The results for the hazard model can be found in Appendix A.

5 Estimation and identification

The estimation procedure to recover model parameters follows that of Gowrisankaran and Rysman (2012) and Derdenger (2014). Since the estimation of the box office market depends on the home video value, we estimate the home video market first and then we proceed with the box office market.

We will use $\vec{\alpha}^b$ and $\vec{\alpha}^h$ to denote $(\alpha^{fe,b}, \alpha^{x,b}, \alpha^{p,b})$ and $(\alpha^{fe,h}, \alpha^{x,h}, \alpha^{p,h})$, respectively; these are the vector of all fixed effects and observable characteristics coefficients. We now discuss the identification of the structural parameters $(\vec{\alpha}^h, \vec{\alpha}^b, \beta)$, which requires solving the home video market and using its results to solve the box office market. We do not attempt to estimate β, since it is well known that estimating the discount factor in dynamic decision models is a notoriously difficult task (Gowrisankaran and Rysman 2012, Magnac and Thesmar 2002). The problem in estimating the discount factor is that consumer waiting can be explained by moderate preferences for movies, or by little discounting of the future. Thus, we set $\beta = 0.9995$ on a weekly level (equivalent to 0.974 yearly), leaving $(\vec{\alpha}^h, \vec{\alpha}^b, \phi)$ to estimate.

In order to allow for the comparison between box office and home video utilities, we must identify the scaling parameter ϕ. To do so, we impose the following constraint in estimation:

$$\alpha^{p,b} = \phi \alpha^{p,dvd}. \tag{24}$$

With ϕ identified, we redefine $\vec{\alpha}^b \doteq (\alpha^{fe,h}, \alpha^{x,h})$, since $\alpha^{p,b}$ is not to be directly estimated.

Following Berry et al. (1995) and Gowrisankaran and Rysman (2012), we specify a generalized method of moments (GMM) function

$$G(\vec{\alpha}^h, \vec{\alpha}^b, \phi) = Z' \vec{\xi}(\vec{\alpha}^h, \vec{\alpha}^b, \phi), \tag{25}$$

where $\vec{\xi}(\vec{\alpha}^h, \vec{\alpha}^b, \phi)$ is the stacked vector unobserved characteristics of the box office market (ξ^b_{jt}), DVD market (ξ^{dvd}_{jt}) and Blu-ray market (ξ^{blu}_{jt}), for which the predicted shares equal the observed shares, and Z is a matrix of exogenous instrumental variables. To control for price endogeneity, instrumental variables consist of lagged home video prices and the price difference from the times mean price for the home market, and observed box office prices for the box office market given that this price is assumed exogenous. Note, the explicit assumption given the set of instruments for box office prices is that box office ticket prices are not correlated with unobserved box office movie quality (ξ^b_{jt}) given they do not vary by movie or by run time. We estimate the parameters to satisfy

$$(\hat{\alpha}^h, \hat{\alpha}^b, \hat{\phi}) = \underset{(\vec{\alpha}^h, \vec{\alpha}^b, \phi)}{\arg\min} \left\{ G(\vec{\alpha}^h, \vec{\alpha}^b, \phi)' W G(\vec{\alpha}^h, \vec{\alpha}^b, \phi) \right\}, \tag{26}$$

where W is a weighing matrix. Thus, to estimate $(\vec{\alpha}^h, \vec{\alpha}^b, \phi)$ we must first solve for $\xi(\vec{\alpha}^h, \vec{\alpha}^b, \phi)$, which requires solving for the shares of all markets. We first discuss how to solve for home video shares and then for box office shares. In the following sections, we explain how to obtain $\hat{\alpha}^h, \hat{\alpha}^b(\phi)$ and $\vec{\xi}(\hat{\alpha}^h, \hat{\alpha}^b(\phi), \phi)$ based on an initial guess of ϕ. The optimal value $\hat{\alpha}^h$ is independent of ϕ and can be solved separately. Given a guess for ϕ, one can solve for the optimal $\hat{\alpha}^b(\phi)$ easily, which will depend on the chosen ϕ. Finally, the optimal solution for $\hat{\phi}$ can be obtained by solving a single variable optimization problem, which includes a subproblem that finds $\hat{\alpha}^b(\phi)$:

$$\hat{\phi} = \underset{\phi}{\arg\min} \left\{ G(\hat{\alpha}^h, \hat{\alpha}^b(\phi), \phi)' W G(\hat{\alpha}^h, \hat{\alpha}^b(\phi), \phi \right\}. \tag{27}$$

5.1 Home video shares

The consumer decision problem for the home video market is defined in Section 4.3.2 as the fixed point of the Bellman equation (10), and the market evolution equation (11). We stack the DVD and Blu-ray panel data, to find the vector $\delta^h_{j,t}$ for which the predicted shares ($\hat{s}^h_{jt}$) equals the observed shares ($\hat{s}^h_{jt}$) for each movie and time period; namely,

$$s^h_{j,t} = \hat{s}^h_{j,t}(\delta^h_{j,t}) \quad \forall j, t. \tag{28}$$

The construction of the observed market shares originate from observed sales and the market size for each movie in each period. The initial home video market size is set to the number of U.S. households for DVD discs, which assumes complete market penetration of DVD players. For Blu-ray discs, the market size is set to the cumulative number of Blu-ray players sold up to period t. In recovering the market size for subsequent periods, we subtract all sales until such period from the initial market size.

Following Gowrisankaran and Rysman (2012) and Berry et al. (1995) the solution to equation (28) can be solved using a fixed point iteration:

$$\delta^{h.new}_{j,t} = \delta^{h.old}_{j,t} + \psi^h \cdot \left(\log(s^h_{j,t}) - \log\left(\hat{s}^h_{j,t}(\delta^h_{j,t})\right)\right), \tag{29}$$

where ψ^h is a tuning parameter set to 0.6.

We now summarize the procedure to solve the home video consumer's problem and obtain the vector $\delta^h_{j,t}$ that simultaneously satisfies equations (10) and (11), with the predicted shares in Equation (12) equal to the observed shares. It is important to remark that this procedure is done separately for DVDs and Blu-rays.

1. Initialize $\delta^h_{j,t} = \log(s^h_{j,t} + 0.00001)$.
2. Define the 25-point vector: $V^{temp}_i = 1$ for $i \in \{1, \ldots, 25\}$.
3. Find OLS estimates of AR(1) parameters ν^{hv}_1 and ν^{hv}_2 following Equation (11).
4. Perform a 25-point Gaussian Quadrature procedure on $\delta^h_{j,t}$ using the estimates ν^{hv}_1 and ν^{hv}_2 to find a discretized grid, Z_i for $i \in \{1, \ldots, 25\}$, for $\delta^h_{j,t}$ as well as the transition probability matrix across the grid, M_z.
5. Set $V^h(Z_i) \leftarrow V^{temp}_i \; \forall i \in \{1, \ldots, 25\}$.

6. Loop $V^h(Z_i) \leftarrow \log\left(\exp(Z_i) + \exp(\beta \mathbb{E}[V^h(Z_i')|Z_i])\right)$ until a fixed point on $V^h(Z_i)$ is reached (tolerance set to 10^{-10}), where the Gaussian Quadrature grid and transition probabilities are used to compute $\beta \mathbb{E}[V^h(Z_i')|Z_i]$ $\forall i \in \{1, \dots, 25\}$, and Z_i' is the transition state from Z_i.

7. Interpolate the vector $\delta^h_{j,t}$ on Z_i and $V^h(Z_i)$ to obtain estimates of $V^h(\delta^h_{j,t})$, and compute the predicted marketshares, $\hat{s}^h_{j,t}$, using Equation (12), and set $V_i^{temp} \leftarrow V^h(Z_i)$.

8. Set $\delta^h_{j,t} \leftarrow \delta^h_{j,t} + \psi^h \cdot \left(\log(s^h_{j,t}) - \log\left(\hat{s}^h_{j,t}\right)\right)$ with $\psi^h = 0.6$, and go to step 3 until a fixed point on $\delta^h_{j,t}$ with a tolerance of 10^{-10} is reached.

After finding the output $\delta^h_{j,t}$ vector that comes from the procedure above (satisfying the Bellman equation (10), the market evolution equation (11), and making the predicted shares equal the observed shares), we save the home video value for each movie j and medium k, DVD or Blu-ray, at the beginning of this market. We then use this value, $V^h_{k,j}$, as the terminal continuation value for the box office market as used in equation (15).

5.2 Box office shares

Similar to the above home video market, market share is determined by specifying the movie's potential market size in each period t. Additionally, the market size for subsequent periods is calculated by subtracting all sales prior to period t from the initial market size. For the box office market, the initial market size for each movie is set at the U.S. population within the age segment of the movie's rating (e.g. PG13 movies would include the number of people older than 13).

Once the home video market is solved, the home video values enter the terminal continuation values in the box office market and we seek a fixed point between equations (17), (18), (19), and (20). This is done for DVD and Blu-ray player owners separately, and then we use (21) and (22) to compute the predicted box office market shares.

As in the home video market, we wish to find a vector $\delta^b_{j,t}$ for which the predicted shares equals the observed shares for each movie and time period; namely,

$$s^b_{j,t} = \hat{s}^b_{jt}(\delta^b_{j,t}) \quad \forall j, t. \tag{30}$$

We solve (30) by iterating over

$$\delta^{b.new}_{jt} = \delta^{b.old}_{jt} + \psi^{box} \cdot \left(\log(s^b_{j,t}) - \log\left(\hat{s}^b_{j,t}(\delta^b_{j,t})\right)\right), \tag{31}$$

where ψ^h is a tuning parameter set to 0.6.

We now summarize the procedure to solve the box office consumer's problem and obtain the vector $\delta^b_{j,t}$ that simultaneously satisfies equations (17), (18), (19), and (20), with the predicted shares in Equation (22) equal to the observed shares.

1. Initialize $\delta^b_{j,t} = \log(s^b_{j,t} + 0.00001)$.

2. Define the 25-point vector: $V_i^{temp} = 1$ for $i \in \{1, \ldots, 25\}$.

3. Find OLS estimates of AR(1) parameters ν_1^{box} and ν_2^{box} following Equation (20).

4. Perform a 25-point Gaussian Quadrature procedure on $\delta^b_{j,t}$ using the estimates ν_1^{box} and ν_2^{box} to find a discretized grid, Z_i for $i \in \{1, \ldots, 25\}$, for $\delta^b_{j,t}$ as well as the transition probability matrix across the grid, M_z.

5. For all $T \in \{1, \ldots, 51\}$ set $V^b_{T,k|T}(Z_i) = \log\big(\exp(Z_i) + \exp(\beta^{NT_{k,j}} V^h_{k,j}\big)$.

6. Perform a backwards induction procedure using the transition probabilities to set $V^b_{t,k|T}(Z_i) \leftarrow \log\big(\exp(Z_i) + \exp(\beta \mathbb{E}[V^b_{t+1,k|T}(Z_i')|Z_i])\big)$ for all $t \in \{1, \ldots, T-1\}$, $T \in \{2, \ldots, 51\}$ and $k \in \{\text{DVD, Blu-ray}\}$.

7. Interpolate the vector $\delta^b_{j,t}$ on Z_i and $V^b_{t,k|T}(Z_i)$ for all $t \in \{1, \ldots, T-1\}$, $T \in \{2, \ldots, 51\}$ and $k \in \{\text{DVD, Blu-ray}\}$.

8. Use the discrete time proportional hazard model presented in section 4.3.3 to compute $V^b_{t,k}(\delta^b_{j,t}, r_t)$ using Equation (19).

9. Compute the predicted marketshares, $\hat{s}^b_{j,t}$, using Equation (22) and set $\delta^b_{j,t} \leftarrow \delta^b_{j,t} + \psi^b \cdot \left(\log(s^b_{j,t}) - \log\left(\hat{s}^b_{j,t}\right)\right)$ with $\psi^b = 0.6$, and go to step 3 until a fixed point on $\delta^b_{j,t}$ with a tolerance of 10^{-10} is reached.

5.3 Recovery of $\vec{\xi}$, $\vec{\alpha}^h$ and $\vec{\alpha}^b$

We use the estimated δ^b and δ^h on a set of regressions involving different movie characteristics. We begin by exposing the retrieval of characteristic coefficients for the home video market, and we proceed with the box office market. By the end of this subsection we describe our procedure to generate advertising and poster color covariates.

5.3.1 Home video

To recover the unobserved characteristics ξ^{dvd} and ξ^{blu}, which are required to compute the GMM objective function (27), we regress δ_{hv} as the purchase utility from equations (1) and (3) on a set of covariates. The covariates involve movie-technology specific dummy variables [9] ($\alpha_j^{fe,h}$), age and the squared age of the movie in weeks, goodwill advertising stock, price, and, month and year dummies. The formation of the goodwill advertising stock follows that of Dubé et al. (2005) and it is explained in detail in subsection 5.3.3. For each covariate, we create a new one that is multiplied by a Blu-ray dummy as shown in (3).

Like other studies of market power since Bresnahan (1981), we allow price to be endogenous to unobserved characteristics (ξ^h), but we assume that movie characteristics are exogenous. This assumption is justified when movie characteristics are determined in advance, independently of unobserved ones at the moment the home videos are sold. As it is common in the literature, we use lagged prices and price differences from the mean as instruments in a two stage least squares regression.

This first stage regression to identify the contribution of time-dependent characteristics, must be performed after every fixed point on $\delta_{j,t}^h$ is achieved. This is because we need $\alpha^{p,h}$ to obtain $\alpha^{p,b}$ according to equation (24) in order to obtain the flow utilities for the box office after finding a fixed point in such a market.

A second stage regression of our model can be performed after estimating $\hat{\phi}$ to recover estimates of non-time varying characteristics. This involves regressing the fixed effects obtained in the first stage regression with movie specific characteristics such as the logarithm of the total first three weeks of box office revenue and its squared term, home window and its square term, logarithm of the production budget for the film, distributor dummies, and poster color dummies (subsection 5.3.4 explains in detail the creation of the poster color dummies). Following Nevo (2000), we perform a minimum distance procedure: let $y = (y_1, \ldots, y_J)'$ denote the $J \times 1$ vector of fixed movie-technology coefficients ($\alpha_j^{fe,h}$) from Equation (3); let X be the $J \times K (K < J)$ matrix of movie characteristics, and $\xi^{fe,h}$ is the movie specific deviation of the unobserved characteristics. Then, we have

$$y = X\beta^h + \xi^{fe,h}. \tag{32}$$

We do not make any assumptions on the error variance covariance matrix (Ω) since we can compute

[9] A movie that is both on DVD and Blu-ray will have separate dummy variables for the panel rows that correspond to DVD and Blu-ray.

it from our first stage regression; thus, instead of of using an Ordinary Least Squares (OLS) procedure we perform a Generalized Least Squares (GLS) one. The GLS estimator is defined as

$$\beta^h_{GLS} = \arg\min_b (y - Xb)'\Omega^{-1}(y - Xb), \tag{33}$$

which can be rewritten as $\arg\min_b[\Omega^{-1/2}(y - Xb)]'[\Omega^{-1/2}(y - Xb)]$. This can be seen as an OLS objective function of $\tilde{y} = \tilde{X}b + \tilde{\xi}^{fe,h}$, with $\tilde{y} \doteq \Omega^{-1/2}y$, $\tilde{X} \doteq \Omega^{-1/2}X$ and $\tilde{\xi}^{fe,h} \doteq \Omega^{-1/2}\xi^{fe,h}$. Thus, the GLS estimator can be written as $\hat{\beta}^h_{GLS} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y} = (X'\Omega^{-1}X)^{-1}X\Omega^{-1}y$, with $\hat{\xi}^{fe,h} = \hat{y} - X\hat{\beta}^h_{GLS}$. Furthermore, we can write the variance of the GLS estimator as $VAR(\hat{\beta}^h_{GLS}) = (X'\Omega X)^{-1}$.

5.3.2 Box office

For each value of ϕ, we impose constraint (24) and regress $\hat{\delta}^b_{j,t} \doteq \delta^b_{j,t} - \phi\alpha^{p,dvd}p^b_{j,t}$ as the flow utilities in equation (6), which involves movie specific dummy variables, the logarithm of the first three week revenue[10], age and the squared age of the movie in weeks, goodwill advertising stock, and current month fixed effects. This yields $\xi^b(\phi)$, which allows for the computation of the GMM objective function (27). We refrain from using a two stage least squares regression as used in the home video market, because the endogeneity of price has already been subtracted with the use of the utility scaling parameter.

We then perform a second stage regression that finds the taste components for the movie specific characteristics, that is, release year, logarithm of production budget, distributor dummies, and poster color dummies. We use the same GLS estimator as in the home video second stage regression.

The procedure to compute the GMM objective function and estimate ϕ may be summarized as follows:

1. Recover $\hat{\delta}^b_{j,t} = \delta^b_{j,t} - \phi\alpha^{p,dvd}p^b_{j,t}$ given ϕ.
2. Run a movie fixed effects regression of $\hat{\delta}^b_{j,t}$ on time dependent movie characteristics to estimate $\alpha^{fe,b}_j$ and $\alpha^{x,b}$ from equation. (6)
3. Compute $\xi^b_{j,t}(\vec{\alpha}^b(\phi)) = \hat{\delta}^b_{j,t} - \alpha^{fe,b}_j + \alpha^{x,b}x^b_{j,t}$.
4. Construct $\xi(\vec{\alpha}^h, \vec{\alpha}^b, \phi)$ and compute the objective function of equation (27).

[10]For the first week, we set this covariate as 0, whereas for weeks 2 and 3 we add all revenue in previous weeks until then.

5.3.3 Goodwill advertising stock

Following Dubé et al. (2005), we implement a simple advertising model that captures the "carry-over" of advertising to posterior periods. Let $A_{j,t}$, $g_{j,t}$ and $g^a_{j,t}$ denote the advertising expenditure, goodwill stock and augmented goodwill stock for movie j in period t. The augmented goodwill stock is what enters in the consumer's utility function, and it is increased by advertising over an already present goodwill stock:

$$g^a_{j,t} = g_{j,t} + \psi(A_{j,t}), \tag{34}$$

where ψ is the goodwill production function. Dubé et al. (2005) discuss some possibilities for ψ; we particularly assume that $\psi(x) = \log(1 + x)$. Augmented goodwill stock in period t depreciates over time, with a discount rate $\lambda \in (0, 1)$ per period, and becomes the beginning of goodwill stock in period $t + 1$:

$$g_{j,t+1} = \lambda g^a_{j,t}. \tag{35}$$

Our advertising data show advertising expenditure per movie per month. Given the difficulty of estimating discount rates, we assume $\lambda = 0.75$ as the monthly discount rate. Once we find the augmented goodwill for a given month, we use that amount for all weeks that start during such month.

5.3.4 Poster Colors

In this section, we describe the procedure to apply color theory in consumer preferences for movie posters. Several online articles discuss the importance of color choices in marketing—see Morton (2012), O'Grady (2019) and Hauff (2018)—suggesting that color theory might be used as a persuasion mechanism to increase purchases. Generally, color theory suggests that the use of complementary colors is used to drive sales.

In order to apply color theory in our model, we downloaded poster images with a resolution of 500×750 for each movie in the data using the API at https://www.themoviedb.org/. Following Ivasic-Kos et al. (2014), we extract color information from each movie poster by transforming the RGB values of pixels to HSV (hue/saturation/value) and extracting its hue. Then, we transform the hue from each pixel to a color by using a discretized 12 color palette, and finally we obtain a 12-color spectrum for each movie poster. As an example, Figure 3 shows the hue spectrum obtained for the poster of the movie "Iron Man 2". We can see that this poster makes use of complementary colors—two hues positioned exactly six spaces away from each other—using orange and azure to attract viewers. This a common practice in action pictures; most of them make use of red/orange/yellow explosions and contrast it with some form cyan/azure/blue.

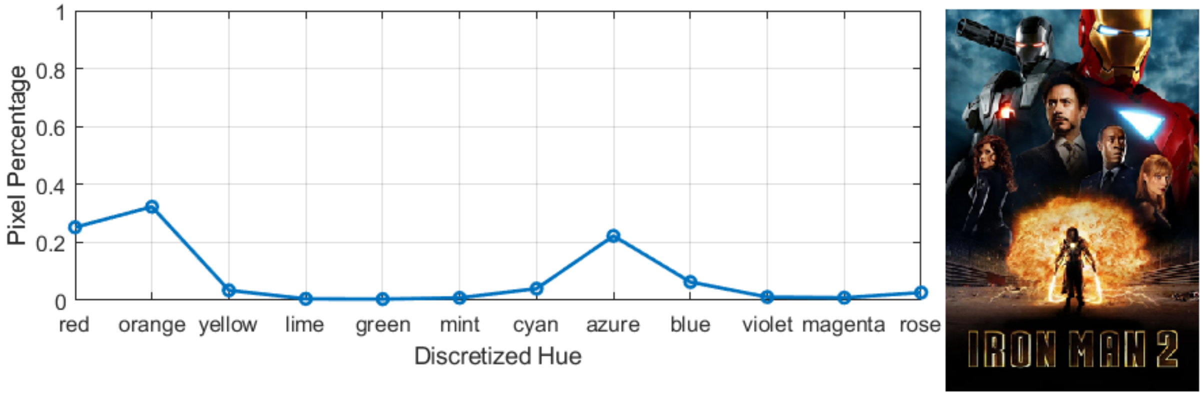

Figure 3: Discretized hue spectrum for the poster of the movie "Iron Man 2" seen on the right.

In order to identify consumer preference for different poster colors and to identify preference for the use of complementary colors, we first create a dummy variable for the color peak in each movie spectrum. We then create two different covariates:

1. peak color strength: we generate it by multiplying a peak color dummy with the percentage of pixels that belong to this color in the poster.

2. complement color strength: we generate it by multiplying a peak color dummy with the percentage of pixels that belong to this color's complement in the poster.

6 Results

We present our time dependent parameter estimates in Table 5. For all markets, the average price coefficient is negative, which means that consumers have marginal disutility towards price. Consumers dislike consuming older products, as seen in the negative values of the age coefficients. Advertising affects utility with a positive effect on all markets, and based on the coefficients it affects Blu-ray purchase utility the most, followed by DVDs and then box office tickets. The "lag 1st3weeks box revenue" variable represents the total box office lagged revenue until a given period or week 4, whichever comes first. The overall contribution of the linear and quadratic logarithmic terms of this variable is increasing with revenue. This means that movies that perform better during the first few weeks since release drive up purchase utility for the upcoming weeks.

Time Dependent Characteristics

Variable	**Box Office**	**Home Video**	**Blu-ray Indicator**
log(lag 1st3weeks box revenue)	0.2356 (0.0459)***	-	-
log(lag 1st3weeks box revenue)^2	-0.0126 (0.0026)***	-	-
Age (weeks)	-0.8158 (0.0157)***	-0.0154 (0.0042)***	-0.0004 (0.0069)
Age^2 (weeks^2)	0.0221 (0.0009)***	0.0000 (0.0000)***	-0.0000 (0.0000)***
Advertising	0.5048 (0.1334)***	0.6645 (0.0009)***	0.1043 (0.0159)***
Price	-0.0113 (-)	-0.0330 (0.002)***	-0.0194 (0.0037)***
ϕ	0.3421 (0.2705)	-	-
Month fixed effects	✓	✓	✓
Year fixed effects		✓	✓
Movie medium fixed effects	✓	✓	✓
N	1,797	70,253	25,373

$^{***}p < .01$, $^{**}p < .05$, $^{*}p < .1$

Table 5: Time dependent parameter estimates.

We present time independent parameter estimates in Table 6. The estimates for the logarithm opening box office revenue show a convex response for the utility, and specifically for the ranges of box office revenue dealt in the data; it is increasing for both DVDs and Blu-rays. This means that the better a movie performs in theaters, the better it will perform in the home video market. The home video window coefficients for DVD and Blu-ray show a concave response to utility. DVDs present a maximum at 5.2 weeks, whereas Blu-rays present a maximum 9.1 weeks. These non-zero maximums could be capturing the effects of word of mouth, and it is important to note that this is the case when everything else is held constant, as the advertising spillover effect from theaters will most likely shift these maximums to lower values. The production budget estimate for the box office and Blu-rays is positive, suggesting that higher production budget films generate greater revenue for the box office and Blu-rays. This effect is opposite for DVDs, showing that increasing the production budget reduces the DVD revenue.

Our results on movie colors are significant and present some interesting features. The use of yellow and its complement in a poster yields the largest contribution to purchase utility for both box office and Blu-rays. This is not surprising, as action movies exhibit explosions and fires in their posters, (which contribute to the yellow hue) and use a complementary color to make these stand out. It is interesting to note that for DVDs, the contribution of yellow and complement is negative, which suggests that DVD consumer behavior is different from the behavior of box office and Blu-ray consumers.

7 Counterfactuals

We use the 113 movies for which we have DVD and Blu-ray sales data. We modify the data by setting the DVD and Blu-ray home windows to a specific number of weeks NT_{dvd} and NT_{blu}, respectively. At the same time, we readjust the advertising goodwill by discounting through longer or shorter periods of time. For each market (box office and home video), we must reach an equilibrium described in Section 4.3, the Consumer's problem. As both markets depend on the box office revenue during the first three weeks of the theatrical run, and the box office market terminal continuation value depends on the home video value, this becomes a challenge. We begin with a guess of the box office revenue during the first three weeks of the theatrical run, and solve the home video market. We use that home video value and box office revenue to solve the box office market. We iterate between both markets until we find a fixed point in box office revenues and home video

Time Independent Characteristics

Variable	**Box Office**	**DVD**	**Blu-ray**
log(1st3weeks box revenue)	-	-3.8676 (0.0884)**	-5.2108 (0.1340)***
log(1st3weeks box revenue)^2	-	0.1226 (0.0027)***	0.1580 (0.0041)***
Home window (weeks)	-	0.0263 (0.0065)***	0.0752 (0.0076)***
Home window^2 (weeks^2)	-	-0.0025 (0.0004)***	-0.0042 (0.0005)***
log(production budget)	0.4902 (0.0324)***	-0.2251 (0.0122)***	0.1652 (0.0187)**
Red indicator × %	0.5099 (0.1055)***	0.1034 (0.0371)***	0.1120 (0.0614)**
Orange indicator × %	0.0635 (0.1146)	0.0797 (0.0384)**	-0.0345 (0.0729)
Yellow indicator × %	-0.3011 (0.2506)	0.2615 (0.0803)***	1.7793 (0.1225)***
Cyan indicator × %	0.5175 (0.1822)***	0.6823 (0.0807)***	1.5314 (0.0985)***
Azure indicator × %	0.2789 (0.0962)***	-0.0409 (0.0404)	0.2504 (0.0618)***
Red indicator × complement%	-2.8862 (0.7920)***	6.1720 (0.3358)***	-0.5325 (0.4289)
Orange indicator × complement%	-1.1732 (0.4195)***	1.0549 (0.1668)***	2.3883 (0.2000)***
Yellow indicator × complement%	14.1576 (3.2520)***	-3.3738 (1.0169)***	21.6641 (1.1852)***
Cyan indicator × complement%	-5.8510 (0.7074)***	-1.31377 (0.2467)***	-1.9363 (0.2581)***
Azure indicator × complement%	-2.6716 (0.3524)***	0.4767 (0.1268)***	0.9212 (0.1793)***
Distributor fixed effects	✓	✓	✓
Release year fixed effects	✓	✓	✓
N	149	149	113

$^{***}p < .01$, $^{**}p < 0.05$, $^{*}p < .1$

Table 6: Time dependent parameter estimates.

values. This must be done for each counterfactual set of values of the home video window in order to evaluate the movie supply chain revenue for different home video windows and optimize around it.

We first analyze the case where both technological qualities, DVD and Blu-ray, are released simultaneously, and then we allow for versioning, where each of them may have different release strategies. We take the point of view of the studios and try to maximize their revenue from both box office and home videos. In order to split the box office revenue between studios and theaters we impose a standard contract (Vogel 2014). This contract involves a house cut of $2,000 per theater per week, and after subtracting the cut the split starts at 70% in favor of the studios, decreasing 10% every two weeks until it reaches 0%. These contracts generate incentives for theaters to exhibit movies for a longer period of time, as the share they keep from box office revenue is greater with the age of the movie.

For each movie, we find the optimal home video window and then we compute the average of these optimal values for reporting. In the case of a simultaneous DVD and Blu-ray release, the maximization problem is the following:

$$\begin{aligned}
\underset{NT \in \mathbb{Z}_{\geq 0}}{\text{maximize}} \quad & \sum_t \Big[M_{j,t}^{box}(NT) \cdot (1-\tau_{jt}) p_{j,t}^{box} \cdot \hat{s}_{j,t}^{box}\big(\delta_{j,t}^{b}(NT)\big) + M_{j,t}^{dvd}(NT) \cdot p_{j,t}^{dvd} \cdot \hat{s}_{j,t}^{dvd}\big(\delta_{j,t}^{h}(NT)\big) \\
& \quad + M_{j,t}^{blu}(NT) \cdot p_{j,t}^{blu} \cdot \hat{s}_{j,t}^{blu}\big(\delta_{j,t}^{h}(NT)\big)\Big] \\
\text{subject to} \quad & (17),\ (18),\ (19),\ \text{and } (20) \quad \forall j \text{ and } t \in [1, \ldots, T-1], \\
& (10), \ \text{ and } (11) \quad \forall\, j \text{ and } t.
\end{aligned}$$

where $M_{j,t}^{m}(NT)$ represents the market size for medium m and movie j at time period t, $\hat{s}_{j,t}^{box}$ are the predicted box office market shares following equation (22), $\hat{s}_{j,t}^{dvd}$ and $\hat{s}_{j,t}^{blu}$ are the predicted market shares for DVD and Blu-ray, respectively, following equation (12), and $\tau_{j,t}$ represents the revenue cut kept by theaters for movie j in period t. The specification for $\delta_{j,t}^{h}(NT)$ and $\delta_{j,t}^{b}(NT)$ follow (9) and (14) , respectively, using the parameter estimates presented in Section 6 and adjusting the goodwill advertising stock vector for each NT value (according to Section 5.3.3). When versioning is allowed, the previous optimization problem changes to one in which the maximization variables become (NT_{dvd}, NT_{blu}) with one additional constraint that is described in Section 7.2.

We now summarize the counterfactual procedure after having estimated the model. The goal of this procedure is to find equilibrium home video values and box office revenues after changing the home video windows. This procedure has to be ran each time the objective function is to be evaluated.

1. Set the counterfactual home video window lengths for all movies to NT_{DVD} and $NT_{\text{Blu-ray}}$ for DVDs and Blu-rays respectively.

2. Initialize the counterfactual box office revenue ($r_{j,t}$) with the data box office revenue, and the counterfactual home video values ($V^h_{k,j}$) with the estimation home video values.

3. Compute the $\delta^h_{j,t}$ vector using the home video counterfactual flow utilities $f^h_{j,t}$ from Equation (3).

4. Do steps 3 through 7 of the procedure described in section 5.1 to obtain the new home video values (${V'_{k,j}}^h$), set $\Delta_V \leftarrow \max_{k,j}({V'_{k,j}}^h - V^h_{k,j})$ and update the counterfactual home video values according to $V^h_{k,j} \leftarrow V^h_{k,j} + \psi \cdot ({V'_{k,j}}^h - V^h_{k,j})$, where $\psi = 0.75$.

5. Compute the $\delta^b_{j,t}$ vector using the box office counterfactual flow utilities $f^b_{j,t}$ from Equation (6).

6. Do steps 3 through 8 of the procedure described in section 5.2 and compute the counterfactual box office marketshares to obtain a new box office revenue ($r'_{j,t}$).

7. Set $\Delta_{box} \leftarrow \max_{j,t}({r'_{j,t}}^h - r_{j,t})$ and update the counterfactual box office revenue to $r_{j,t} \leftarrow r_{j,t} + \psi \cdot (r'_{j,t} - r_{j,t})$.

8. Go to step 3, unless $\Delta_V \leq 10^{-10}$ and $\Delta_{box} < 1$.

7.1 Simultaneous DVD and Blu-ray Release

We conduct the counterfactual analysis in which the DVDs and Blu-rays are released simultaneously, as this was the current industry practice during the data time period. Figure 4 shows a histogram of the home video windows for the data, and for the studio optimal values. As we see, for most movies it is optimal to shrink the home video window to a range from 0 to 4 weeks. This means that advertising spillover effect from theaters and the home video freshness dominate the increased competition to theaters that an early home video release may provide. Furthermore, the demand cannibalization of the box office, generated by this early release, reduces the box office signal, which could impact home videos, but again the advertising spillover effect dominates.

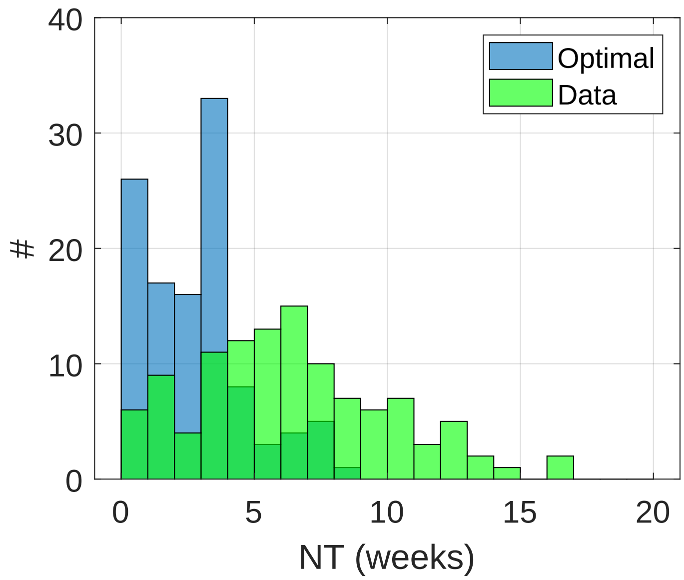

Figure 4: Histogram of the home window lengths, for the data and the optimal.

Figure 5 shows the studio revenue from the box office and home videos for a particular movie as a function of the home video window. We see a steep increase in the home video revenue from shrinking the home video window, whereas the demand cannibalization to theaters is minimal. Table 7 shows the average home video window, studio revenue, and theater revenue, under the optimized strategy and the data. We can see that the average optimal strategy shrinks the home video window to about 2.3 weeks, providing an increased studio revenue of 4.47% and an increase in theater revenue of 0.08%[11].

Simultaneous Release Revenues

Averages	**NT (weeks)**	**Studio Revenue ($)**	**Theater Revenue ($)**
Optimal Simultaneous Release	2.3186	124.34M (+4.47%)	64.59M (+0.08%)
Data	6.1960	119.02M	64.45M

Table 7: Home video windows and revenue information for the optimal simultaneous release strategy and the data. Studio revenue accounts for the home video window as well as the box office revenue studio share, while theater revenue accounts for the portion or box office revenue that corresponds to theaters.

[11]Such a small increase in theater revenue is within the error of this procedure.

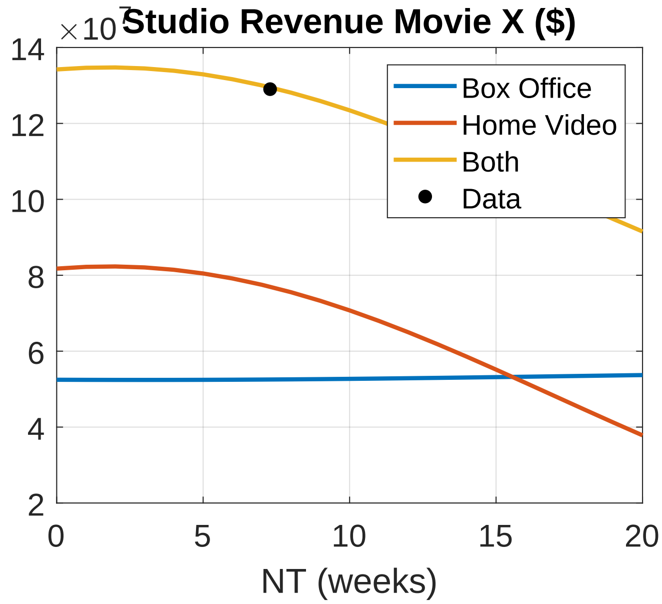

Figure 5: Studio revenue from the box office, home videos, and both, as a function of the home video window for a particular movie. The gray circle represents the studio revenue at the data home video window of 7 weeks.

In the following section, we allow for separation between DVD and Blu-ray releases and analyze its benefits.

7.2 Versioning: Separate DVD and Blu-ray releases.

We now analyze the counterfactual in which the home video window may differ for different technological qualities. We denote NT_{dvd} and NT_{blu} as the home video window for DVD and Blu-ray, respectively.

In order to ensure that Blu-ray player owners benefit from choosing a Blu-ray over a DVD for each movie, we must impose an incentive compatibility constraint. In general, the value of the Blu-ray is larger than the value of the DVD, but if the Blu-ray release were to be delayed, some consumers might switch to the DVD. By imposing this constraint we ensure that our market size for Blu-rays would be better-off choosing Blu-rays over DVDs. This constraint is as follows:

$$\beta^{NT_{dvd}} V_{dvd} \leq \beta^{NT_{blu}} V_{blu}; \tag{36}$$

the discounted ex-ante value of the Blu-ray has to be greater than or equal to the discounted ex-ante value of the DVD. This is imposing an upper bound on the difference between home video windows. We can rewrite constraint (36) to

$$NT_{blu} - NT_{dvd} \leq \frac{\log\left(\frac{V_{blu}}{V_{dvd}}\right)}{|\log \beta|}. \tag{37}$$

We now search over (NT_{dvd}, NT_{blu}) to find the optimal studio revenue satisfying the incentive compatibility constraint (37). Figure 6 shows the histogram of home video windows for the data, the DVD optimal, and the Blu-ray optimal. We can see a clear difference with the simultaneous release from Figure 4. Now it is optimal to have an immediate after theater release for DVDs, whereas it is optimal to delay the Blu-ray release to an average of about 5 weeks. We see that for DVDs, the advertising spillover from theaters offsets the increased competition. For Blu-rays, these two effects are more balanced. This happens because the ex-ante value function for DVDs is smaller than the one for Blu-rays, so shrinking the home video window for DVDs has very little effect on box office purchases. Then managers can shrink the window and reap the benefits of the advertising spillover effect. This result supports the idea that higher technological quality home videos are closer substitutes to the box office than lower quality home videos. Shrinking the home video window for Blu-rays will not only impact the box office demand, but it will also reduce the box office revenue, reducing the movie quality signal for the home video market (DVD and Blu-ray).

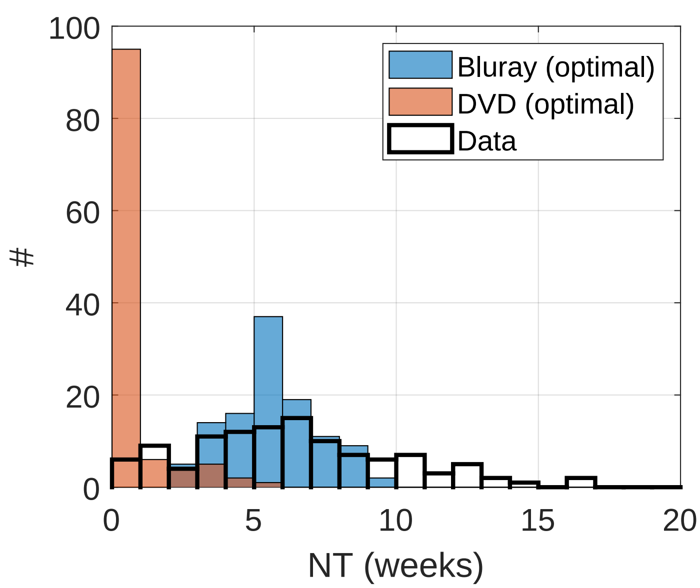

Figure 6: Histogram of the home window lengths, for the data, and the optimal ones for DVD and Blu-ray.

Figure 7 shows the optimal studio revenue as a function of the DVD and Blu-ray home video windows for the same movie as in Figure 5. We clearly see that there is a greater steepness in the DVD home video window axis compared to that of the Blu-ray one. This is because DVD and theaters experience very little competition between each other, and shortening the home video window allows DVD to benefit from the advertising spillover effect. For Blu-rays, however, we see lower steepness in the revenue because the trade-off is more balanced. Table 8 illustrates the new average optimal release windows, studio revenues, and theater revenues. We can see that the average optimal DVD and Blu-ray windows are about 0.4 and 5.2 weeks, respectively. The studio revenue increases almost an extra 1% with respect to the simultaneous release in Table 7, whereas the theater revenue remains almost the same. This result highlights the benefit of exploiting market segmentation strategies in a consumer base that expresses heterogeneous preferences on home video technological qualities. Further improvements could be made by optimizing over advertising periods, and home video pricing, but that is outside the scope of this paper.

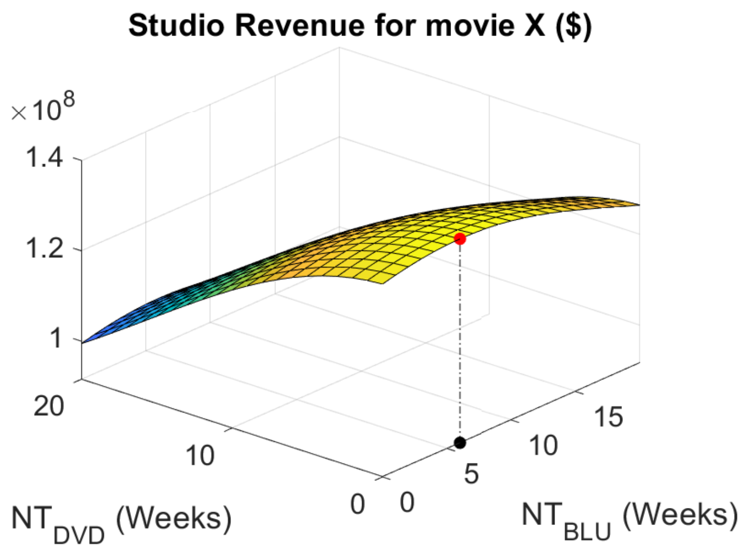

Figure 7: Studio revenue from the box office, home videos, and both, as a function of the home video window for a particular movie. The gray circle represents the studio revenue at the data home video window of 7 weeks.

8 Conclusion

In this paper we build and estimate a dynamic discrete choice model about movie distribution involving the box office, DVDs, and Blu-rays. We capture consumers' value for delaying purchase decisions, allowing for substitution between box office tickets and home videos. We run counter-

Separate Release Revenues

Averages	**NT (weeks)**	**Studio Revenue ($)**	**Theater Revenue ($)**
Optimal Release (DVD,Blu-ray)	(0.3717, 5.1504)	125.40M (+5.36%)	64.56M (+0.03%)
Data	6.1960	119.02M	64.45M

Table 8: Home video windows and revenue information for the optimal separate release strategies and the data. Studio revenue accounts for the home video window as well as the box office revenue studio share, while theater revenue accounts for the portion or box office revenue that corresponds to theaters.

factual simulations in order to determine the optimal the home video window by optimizing studio revenue for each movie and we find that releasing DVDs 0.4 weeks after the theatrical run, and Blu-rays about 5.2 weeks after, is optimal on average. This strategy achieves an average revenue increase of 5.36% for the studios with respect to the current practice, while having minimal impact on theaters. This analysis suggests that higher technological quality home videos are closer substitutes to theater, and their release balances advertising spillover effect from theaters with demand cannibalization. Releasing higher technological quality home videos early cannibalizes theater demand, which impacts theatrical revenue and further impacts all home video markets. For lower technological quality home videos such as DVDs, this is not the case, as they don't compete with theaters; releasing them early reaps all the benefits of the advertising spillover effect. These results highlight the benefit of exploiting market segmentation strategies in a consumer base that expresses heterogeneous preferences on home video technological qualities. We expect that our approach can be carried over to the analysis of the "streaming (HD vs. 4K) versus theaters" debate.

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A Hazard Model Results

In this appendix we report the parameter estimates for the discrete time hazard model presented in Section 4.3.3. We set $M = 51$ and used data from over $1,500$ movies across different genres to find the maximum likelihood estimates. We did because we assume that the decision on whether to remove a movie from theaters or not does not depend on the movie genre, but only on its performance. This way we can take advantage of several extra data points to have better maximum likelihood estimates.

The maximum likelihood estimates for β was -0.1359, whose negative sign decreases the hazard probability in (23) as the movie revenue increases. The maximum likelihood parameters for the λ_{0i} vector is reported as a bar graph in Figure 8.

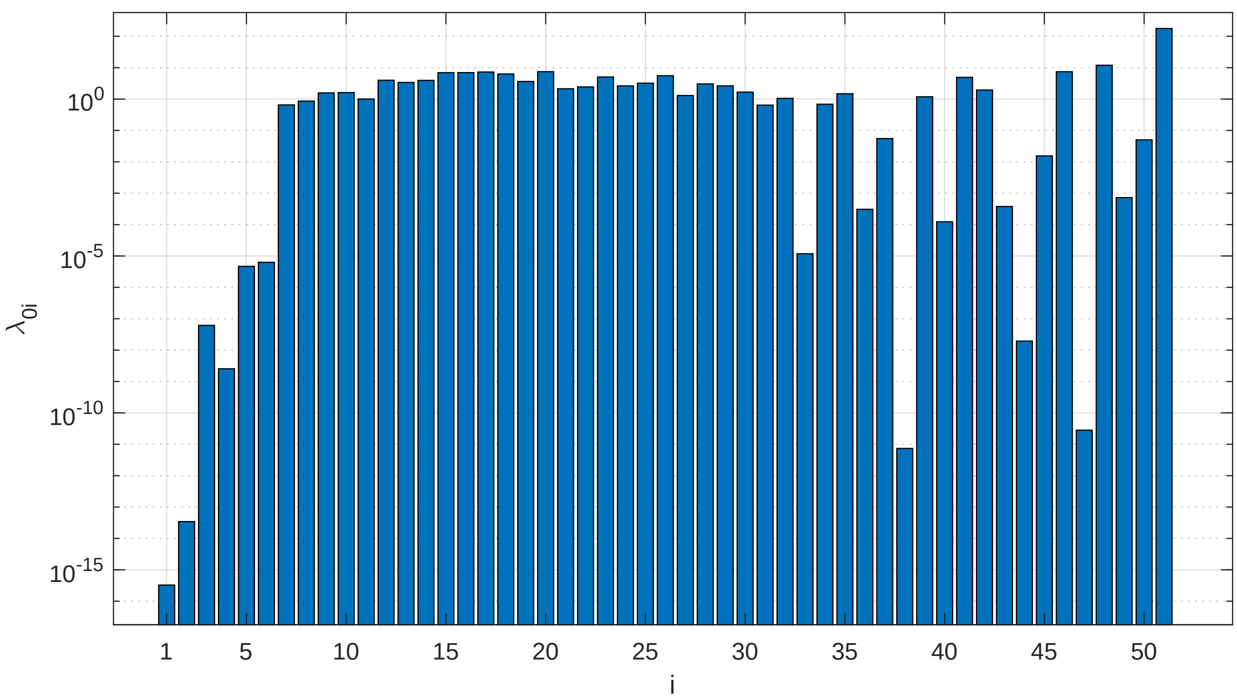

Figure 8: Maximum likelihood estimates for the λ_{0i} vector.